APPENDIX

Poisson prior distribution for the population size, N.

$$P(N=k)=\frac{e^{-m}m^k}{k!}\text{, for } k = 0, 1, 2, \ldots$$

That these probabilities must add to unity provides the following expansion:

$$e^m=\sum_{k=0}^{\infty}\frac{m^k}{k!},$$

valid for any m. The probability that a survey effectively covering area E out of a historical range with area A will detect $X = x$ individuals from the population is the binomial probability:

$$p\left(X=x\mid N,E,A\right)=\frac{N!}{x!(N-x)!}\left(\frac{E}{A}\right)^x\left(1-\frac{E}{A}\right)^{(N-x)},\ \text{ for } x = 0, 1,\ldots, N.$$

This assumes each bird has probability E/A of detection and that birds are detected independently. In particular, the chance that no birds are detected is:

$$P(X=0\mid N,E,A)=\left(1-\frac{E}{A}\right)^N.$$

The posterior distribution of N given no birds are detected is:

$$P(N=k\mid X=0,E,A)=\frac{P(X=0\mid N,E,A)\,P(N=k)}{P(X=0\mid E,A)}.$$

The denominator is the sum over k of the numerator values:

$$\begin{aligned}P(X=0\mid E,A)&=\sum_{k=0}^{\infty}P(X=0\mid N=k,E,A)\ P(N=k)\\&=e^{-m}\sum\frac{\left[m\left(1-\frac{E}{A}\right)\right]^k}{k!}\\&=e^{-m}e^{m\left(1-\frac{E}{A}\right)}\\&=e^{\frac{-mE}{A}}\end{aligned}$$

by the previous expansion. So

$$P(N = k \mid X = 0, E, A) = \frac{e^{-m\left(1-\frac{E}{A}\right)}\left[m\left(1-\frac{E}{A}\right)\right]^k}{k!},$$

which is a Poisson distribution with mean $m(1 - E/A)$. In particular,

$$P(N = 0 \mid X = 0, E, A) = e^{-m\left(1-\frac{E}{A}\right)}.$$

Geometric prior distribution for the population size, N

$$P(N = k) = (1 - p)p^k, \text{ for } k = 0, 1, \ldots$$

That these probabilities add to unity establishes the expansion:

$$\frac{1}{(1-p)} = \sum_{k=0}^{\infty} p^k,$$

valid for any p between zero and one. The derivation of the posterior distribution proceeds here exactly as above for the posterior with the Poisson prior distribution, resulting in

$$P(N = k \mid X = 0, E, A) = \left[1 - p\left(1-\frac{E}{A}\right)\right]\left[1 - p\left(1-\frac{E}{A}\right)\right]^k, \text{ for } k = 0, 1, \ldots$$

which is a geometric distribution with parameter $p(1 - E/A)$. The posterior probability of extinction is therefore

$$P(N = 0 \mid X = 0, E, A) = 1 - p\left(1-\frac{E}{A}\right).$$